Narrow muon bundles from muon pair production in rock

V.A. Kudryavtsev [a,1], E.V. Korolkova [b], N.J.C. Spooner [a]

[a] *Department of Physics and Astronomy, University of Sheffield, Sheffield S3 7RH, UK*
[b] *Institute for Nuclear Research of the Russian Academy of Science, Moscow 117312, Russia*

Abstract

We revise the process of muon pair production by high-energy muons in rock using the recently published cross-section. The three-dimensional Monte Carlo code MUSIC has been used to obtain the characteristics of the muon bundles initiated via this process. We have compared them with those of conventional muon bundles initiated in the atmosphere and shown that large underground detectors, capable of collecting hundreds of thousands of multiple muon events, can discriminate statistically muon induced bundles from conventional ones. However, we find that the enhancement of the measured muon decoherence function over that predicted at small distances, recently reported by the MACRO experiment, cannot be explained by the effect of muon pair production alone, unless its cross-section is underestimated by a factor of 3. © 1999 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Muon bundles, or events with muon multiplicity more than 1, are studied in underground experiments to obtain information on the primary cosmic-ray composition and characteristics of hadron-nucleus interactions. Conventional techniques include comparison of the measured multiplicity and pair separation distributions with predictions based on primary composition models and models of the development of hadronic cascades in the atmosphere. There is, however, an effect which can slightly modify such distributions. This is the process of muon pair production by muons in the rock (or water) above the detector. The cross-section of the process is quite small but its effect is not negligible for small separation distances between muons in the bundles. Double or triple muon events observed in underground experiments can arise from either multiple muon production in Extensive Air Showers (EAS) in the atmosphere or muon pair production by single muons in the rock (a double muon event is observed if one of the muons is stopped on its way to the detector).

---

[1] E-mail: v.kudryavtsev@sheffield.ac.uk

As single muons dominate over multiple muons underground, muon bundles produced by single muons in the rock can contribute to the total number of bundles. An excess in the number of detected bundles over the predictions of EAS models can then be visible. Muon bundles produced in water are the background in underwater detectors looking for up-going neutrino-induced muons [1–4].

Original estimates of the fluxes of muon bundles from muon pair production using one-dimensional calculations have been published in [5]. First three-dimensional simulations of the muon transport through standard rock taking into account muon pair production have been performed recently [6]. In both cases authors used the cross-section calculated in [7] and parametrized in [8]. In [6] it was shown that the process should contribute to the number of narrow muon bundles detected in underground experiments, such as LVD [9] and MACRO [10]. The observation of this effect has been recently reported by the MACRO collaboration [11]. However, the cross-section from [7,8] used in [6] was obtained assuming point-like nuclei with complete screening, which is not a reasonable approximation in this case (see [12] for the discussion). Since then, a new, more accurate cross-section for muon pair production by muons has been obtained [12]. The cross-section calculated in [12] is roughly 2–5 times smaller than that of [7,8]. The new cross-section has been used by the authors of [12] to calculate total fluxes of muon bundles produced in the rock for different depths of observation underground using an one-dimensional numerical integration technique. However, the total fluxes of muon-induced bundles are hidden by the two-orders of magnitude higher flux of conventional muon bundles initiated in EAS in the atmosphere. Muon bundles from muon pair production can be discriminated only by the characteristic that they have small separation between muons in the bundle [6] (later on we will call them narrow muon bundles). Small muon pair separation in such events arises because they are produced in the rock quite near the observation level, while conventional muon bundles are produced in the upper layers of the atmosphere. Thus, a full three-dimensional Monte Carlo simulation is necessary to evaluate the effect caused by muon pair production and to give predictions for underground experiments.

In this paper we describe such simulations using the new cross-section [12] and discuss the implications of the results for underground experiments. We show that the excess of narrow muon bundles, caused by their production in rock, over predictions from EAS models is visible and can be measured experimentally. Such measurements performed at various depths underground could be used as a test of the cross-section at different energies. Our calculations differ from those in [6] by the use of the new, more accurate, cross-section of muon pair production by muons from [12]. They differ from the semi-analytical approach in [12] by the use of a full three-dimensional Monte Carlo technique which is the only way to give predictions for muon separation in the bundles.

2. Muon transport through rock and water

The three-dimensional Monte Carlo code MUSIC [13] is designed to simulate propagation of muons from sea level down to the level of observation. The code takes into account the stochasticity of all processes with fractional energy transfer $v > 10^{-3}$. It calculates the angular deviation and lateral displacement of muons due to multiple scattering and stochastic processes. The code is regularly updated [14] and includes the most recent and accurate cross-sections of muon interactions in matter: bremsstrahlung [15], electron-positron pair production [16] (with the corresponding cross-section on atomic electrons from [17]), and inelastic scattering [18].

The latest version of the code has been modified to take into account muon pair production with fractional energy transfer $v > 10^{-3}$ with the cross-section from [12]. An analytical expression for the differential cross-section as a function of final muon energies was obtained [12] by analogy with similar cross-section for electron pair production by muons [16]. Comparison with precise numerical integration done in [12] showed that the accuracy of parametrization is better than 10% for initial muon energy greater than 10 GeV and final muon energies greater than 1 GeV, while the accuracy of the total cross-section is better than 3% at initial muon ener-

gies greater than 30 GeV. Effects of nucleus screening and finite nucleus size were taken into account in the calculations [12].

We have not restricted the simulation to only one interaction with muon pair production (as had been done in [6]), but considered all possible interactions which, in principle, can result in a muon bundle with multiplicity more than 3.

The differential cross-section as a function of the scattering angle of each muon is not available. Therefore, the scattering of muons at the point of muon pair production has not been taken into account. However, in general, the scattering of muons due to stochastic processes can be neglected, because the resulting angular deviation and lateral displacement are fully determined by the multiple Coulomb scattering [13].

We want to emphasize also that precise simulation of the muon bundles initiated in EAS in the atmosphere is out of the aims of this paper. To estimate the significance of the effect caused by muon pair production in rock, we will compare the calculated flux of muon bundles from this process with the flux of muon bundles measured in underground experiments. To calculate the flux of muon-induced bundles it is enough to use a power-law parametrization of muon energy spectrum at the surface without detailed knowledge of muon multiplicity and separation in a particular shower. This is because single muons dominate over multiple muons by almost one order of magnitude (for existing underground detectors) and, hence, single muons will provide major contribution to the effect mentioned above.

For comparison we have carried out the simulations both in standard rock and in water. 10^7 single muons with an initial energy at sea level sampled according to a power-law spectrum with power index -3.7, were propagated down to different depths in rock and water. A lower edge of the spectrum at sea level was chosen for each depth separately to ensure a muon survival probability ≤ 0.003. This means that all muons which have a probability of more than 0.003 of reaching a predefined depth, have been included in the analysis. Note that the muon energy spectrum at sea level has a more complicated form than a simple power-law at energies below 1 TeV and, hence, the results for shallow depths may be

biased. However, the purpose of this work is not to provide exact estimates of this effect, which is dependent on the details of individual experiments, but only to evaluate the significance of the effect and to show how it can be measured.

3. Results and discussion

The results of the simulations are presented in Table 1. Mean survival probabilities for single muons (N_{surv}/N_{sim} in Table 1) agree well with earlier calculations [6] and with the results of the original code [13]. The ratios of the total numbers of narrow muon bundles to that of single muons (R_2, R_3 and R_b in Table 1) are significantly smaller (by a factor of approximately 3 at 3 km w.e.) than those obtained in [6] due to the different muon pair production cross-section used but agree well with semi-analytical calculations [12] (see also Fig. 1 for comparison with the results from [12]). Table 1 shows also the main characteristics for narrow bundles and single muons, such as: mean energy of single muons $\langle E_s \rangle$, mean energy of muons in bundles $\langle E_b \rangle$, mean path of muon bundle in the rock/water between the point of its production and the observation level $\langle L \rangle$, mean muon pair separation in double ($\langle D_2 \rangle$), triple ($\langle D_3 \rangle$), and double + triple ($\langle D_b \rangle$) muon events, mean angular separation of muons in bundles $\langle \alpha \rangle$, and the ratio of number of pairs with separation less than 1 m to the number of single muons $R_b(<1\text{ m})$. The last column shows the results for 3 km of water. To calculate mean pair separation, mean angular separation of muons and the number of muon pairs we applied weighting $1/N_p$, where N_p is the number of independent muon pairs in the bundle: $N_p = 1$ for double muon events and $N_p = 3$ for triple muon events. This has been done to allow easy comparison between our simulations and results of studies of muon bundles detected in underground experiments (see, for example, [9,11]). Only muons with energy more than 1 GeV (a typical threshold in underground experiments) have been taken into account. The ratios of the number of muons bundles to that of single muons together with the results of [12] are shown also in Fig. 1.

As can be seen in Table 1 and Fig. 1, the ratios of narrow muon bundles to single muons (R_2, R_3 and

Table 1

Main characteristics of narrow muon bundles for different depths in standard rock (columns 2–5) and water (column 6) (1 km w.e. $= 10^5$ g/cm^2): number of simulated muons N_{sim}, number of single muons survived N_{surv}, ratio of the total number of double (R_2) triple (R_3) and double + triple (R_b) muon events to that of single muons, mean energy of single muons $\langle E_s \rangle$, mean energy of muons in bundles $\langle E_b \rangle$, mean path of muon bundle in rock/water between the point of its production and the observation level $\langle L \rangle$, weighted mean separation of muon pairs in double ($\langle D_2 \rangle$), triple ($\langle D_3 \rangle$), and double + triple ($\langle D_b \rangle$) muon events, weighted mean angular separation of muon pairs in bundles $\langle \alpha \rangle$, and the ratio of weighted number of pairs with separation distances less than 1 m to the number of single muons $R_b(< 1\,m)$

	Depth, km w.e.				3 km of water
	1	3	5	10	
N_{sim}	10^7	10^7	10^7	10^7	10^7
N_{surv}	$4.01 \cdot 10^6$	$3.41 \cdot 10^6$	$1.94 \cdot 10^6$	$4.12 \cdot 10^6$	$3.47 \cdot 10^6$
R_2	$5.54 \cdot 10^{-5}$	$1.81 \cdot 10^{-4}$	$2.69 \cdot 10^{-4}$	$4.03 \cdot 10^{-4}$	$1.48 \cdot 10^{-4}$
R_3	$2.07 \cdot 10^{-5}$	$7.21 \cdot 10^{-5}$	$8.81 \cdot 10^{-5}$	$1.04 \cdot 10^{-4}$	$5.04 \cdot 10^{-5}$
R_b	$7.61 \cdot 10^{-5}$	$2.53 \cdot 10^{-4}$	$3.58 \cdot 10^{-4}$	$5.08 \cdot 10^{-4}$	$1.99 \cdot 10^{-4}$
$\langle E_s \rangle$, GeV	88	247	314	366	321
$\langle E_b \rangle$, GeV	221	386	545	793	513
$\langle L \rangle$, km w.e.	0.256	0.585	1.001	2.032	0.653
$\langle D_2 \rangle$, cm	49	81	93	110	165
$\langle D_3 \rangle$, cm	29	33	46	69	77
$\langle D_b \rangle$, cm	44	68	81	101	143
$\langle \alpha \rangle$, deg	1.11	0.893	0.757	0.634	0.615
$R_b(< 1\,m)$	$6.78 \cdot 10^{-5}$	$1.94 \cdot 10^{-4}$	$2.57 \cdot 10^{-4}$	$3.21 \cdot 10^{-4}$	$1.07 \cdot 10^{-4}$

R_b) increase with depth. This results from the rise in interaction cross-section with muon energy. As the
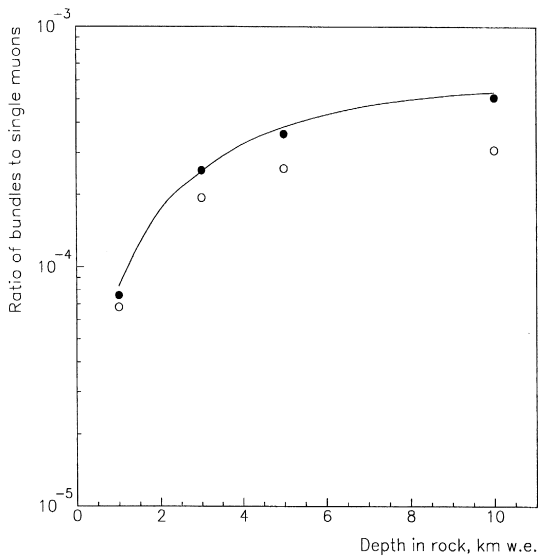

Fig. 1. Ratios of narrow muon bundles to single muons vs. depth in standard rock: *filled circles* – whole sample of muon bundles; *open circles* – weighted pairs with distances less than 1 m between muons; *solid curve* – results of [12] for the whole sample of muon bundles.

mean muon energy which contributes to the muon flux at a particular depth increases with depth, the effective cross-section also increases and hence so does the fraction of narrow muon bundles with respect to single muons. The mean energy of muons in bundles ($\langle E_b \rangle$) is roughly twice that of single muons ($\langle E_s \rangle$) and increases with depth. Muons in bundles cross on average $1/5$ of the total thickness of rock from the point of interaction down to the observation level. The ratio of double to triple muon events (R_2/R_3) increases with depth due to the increase in thickness of rock crossed by the bundle ($\langle L \rangle$) and, hence, due to the decrease in the probability that all three muons survive. The increase in mean muon energy in the bundles partly compensates for this effect.

The weighted mean separation of muon pairs ($\langle D_2 \rangle$, $\langle D_3 \rangle$, $\langle D_b \rangle$) increases by a factor of 2–2.5 with increase of depth from 1 to 10 km w.e., while the weighted mean angular separation for pairs ($\langle \alpha \rangle$) decreases by a similar factor. This can be explained in the following way. Pair separation, determined in this case by multiple Coulomb scattering, depends mainly on the muon path in the rock. As the mean path of muons in bundles down to the observation level ($\langle L \rangle$) increases with depth, pair separation also

increases. The increase in mean muon energy partly compensates for the rise in ($\langle L \rangle$). The angular separation of muons due to Coulomb scattering is determined mainly by the muon energy. Since mean muon energy increases with depth, mean angular separation decreases.

We have found the mean separation of muon pairs to be about 50% higher than obtained in [6]. This is possibly explained by the larger thickness of rock crossed on average by muon bundles in the case of a smaller bundle production cross-section (see also the comparison between values of $\langle L \rangle$ in rock and water in Table 1).

The ratio of muon bundles to single muons in water is smaller than that in the rock. Such behaviour is expected from the Z^2/A-dependence of the macroscopic cross-section on the target nucleus charge and mass. Weighted distributions of pair separation distances for a depth of 3 km w.e. in rock and water are presented in Fig. 2. Due to the lower density of water, the distribution in water is wider than that in the rock.

The small mean pair separation of these bundles can be used to discriminate them statistically from conventional muon bundles originating in Extensive
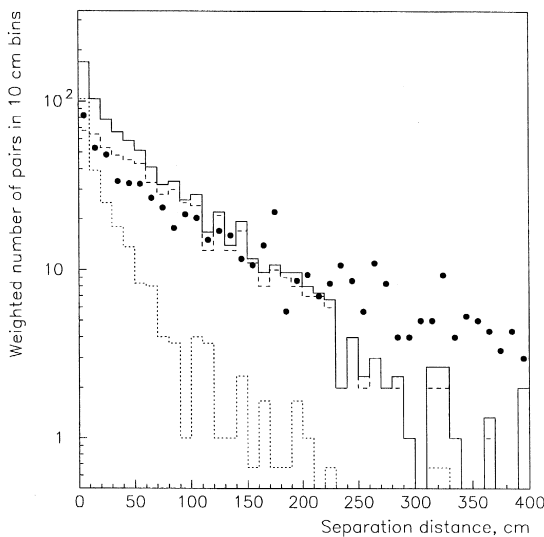

Fig. 2. Weighted distributions of separation of muon pairs in double (dashed histogram), triple (dotted histogram) and all (double + triple) (solid histogram) muon events at 3 km w.e. in rock and the distribution for all muon bundles at 3 km depth in water (filled circles).

Air Showers in the atmosphere. The ratio of conventional muon bundles to single muons is typically (1–10)% depending on the detector geometry. The mean pair separation of conventional muon bundles is of the order of several meters. Only a few percent of muon pairs in conventional muon bundles contribute to separation distances less than 1 m, while from 60% to 90% of pairs in narrow muon bundles have muons separated by less than 1 m. The ratio of the weighted number of pairs with separation distances less than 1 m in narrow bundles to the number of single muons ($R_b(< 1$ m)) in Table 1, see also Fig. 1) is $2.5 \cdot 10^{-4}$ at 3 km w.e. This number has to be compared with $\approx 2 \cdot 10^{-3}$, the estimated ratio of pairs with separation distances less than 1 m in detected bundles to single muons [6]. A 10% effect can easily be found if the total statistics contains hundreds of thousands of muon bundles. This is certainly within the reach of modern underground detectors.

It is of interest to consider the LEP detectors at CERN [19,20]. Despite being located at shallow depth (where the ratio of narrow muon bundles to single muons is quite small), these detectors, having good spatial, angular and energy resolution and the ability to collect millions or even billions of muon events [19], could be a means of searching for the effect above.

Existing deep underground experiments such as MACRO [10] and LVD [9] are also able to search for an excess of narrow muon bundles over the expected number of conventional muon bundles. In fact, evidence for an excess at separation distances less than 3 m has recently been published by the MACRO collaboration [11]. The authors explained the excess as due to production of muon pairs in the rock. However, to calculate the predicted rate they used the cross-section from [8], which gave an overestimate of the effect by a factor of 3. This means that their measured rate should be about factor of 3 higher than expected with the newly calculated cross-section from [12]. Moreover, the measured pair separation distribution (after subtraction of the contribution from conventional muon bundles) is much wider than expected from our simulations. Note, that their calculated mean separation distance is almost twice that of our result for 3 km w.e., which is hard to explain by the difference in the cross-sections

used and the contribution from muons crossing larger thicknesses of rock. Our simple estimates, based on the published pair separation distribution [11] and relative rates of single and multiple muon events [10], show that MACRO observed the ratio of weighted number of pairs with separation distances less than 0.8 m to the number of single muons to be $(7–8) \times 10^{-4}$, while our simulations predict the ratio to be about 2.2×10^{-4}. We conclude that the enhancement of the decoherence function over conventional muon bundle predictions seen by the MACRO experiment cannot be explained by the effect of muon pair production alone, unless its cross-section is underestimated by a factor of 3. Even in this case, it is difficult to explain the difference in the shape of the pair separation distribution.

4. Conclusions

We have revised the process of muon pair production in rock using the recently re-calculated cross-section for this process. Our three-dimensional Monte Carlo of muon propagation through rock shows that there is an observable excess (about 10% at 3 km w.e.) in the number of muon pairs with separation distances less than 1 m over conventional muon bundles, even though the newly calculated cross-section is 2–5 times less than that used before. Observation of this excess is within the reach of existing large underground detectors capable of collecting hundreds of thousands of multiple muon events. However, the excess of muon pairs with separation distances less than 3 m, recently published by the MACRO collaboration, cannot be explained by muon pair production alone unless the cross-section of the process is 3 times larger than that used in this work.

The modified three-dimensional muon propagation code MUSIC, which includes also muon pair production, can be obtained by request to v.kudryavtsev@sheffield.ac.uk

References

[1] S. Hundertmark for the AMANDA Collaboration, in: Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, Utah, USA, 17–25 August, 1999, Vol. 2, p. 9.

[2] I.A. Belolaptikov et al., Astroparticle Phys. 7 (1997) 263.

[3] L. Moscoso for the ANTARES Collaboration, in: Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, Utah, USA, 17–25 August, 1999, Vol. 2, p. 440.

[4] S. Bottai for the NESTOR Collaboration, in: Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, Utah, USA, 17–25 August, 1999, Vol. 2, p. 456.

[5] S.R. Kelner, Yu.D. Kotov, V.M. Logunov. Acta Phys. Acad. Sci. Hungaricae 29, Suppl. 4 (1970) 299; Sov. J. Nucl. Phys. 21 (1975) 763.

[6] V.A. Kudryavtsev, O.G. Ryazhskaya, Il Nuovo Cimento 21C (1998) 171; V.A. Kudryavtsev, O.G. Ryazhskaya, in: Proc. 25th Intern. Cosmic Ray Conf., Durban, South Africa, 28 July – 8 August, 1997, Vol. 6, p. 405.

[7] S.R. Kelner, Sov. J. Nucl. Phys. 5 (1967) 1092.

[8] E.V. Bugaev, Yu.D. Kotov, I.L. Rozental, Cosmic muons and neutrinos, Atomizdat, Moscow, 1970 [in Russian].

[9] M. Aglietta et al., LVD Collaboration, Nucl. Phys. B (Proc. Suppl.) 35 (1994) 243.

[10] M. Ambrosio et al., MACRO Collaboration, Phys. Rev. D 56 (1997) 1407, 1418.

[11] MACRO Collaboration, M. Ambrosio et al., Phys. Rev. D 60 (1999) 032001.

[12] S.R. Kelner, R.P. Kokoulin, A.A. Petrukhin, in: Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, Utah, USA, 17–25 August, 1999, Vol. 2, p. 20.

[13] P. Antonioli, C. Ghetti, E.V. Korolkova, V.A. Kudryavtsev, G. Sartorelli, Astroparticle Phys. 7 (1997) 357.

[14] The code MUSIC is maintained and regularly updated by one of us (VAK) and can be obtained by request to v.kudryavtsev@sheffield.ac.uk.

[15] S.R. Kelner, R.P. Kokoulin, A.A. Petrukhin, Phys. At. Nucl. 60 (1997) 576.

[16] R.P. Kokoulin, A.A. Petrukhin, in: Proc. 12th Intern. Cosmic Ray Conf., Hobart, 1971, Vol. 6, p. 2436.

[17] S.R. Kelner, Phys. At. Nucl. 61 (1998) 448.

[18] L.B. Bezrukov, E.V. Bugaev. Sov. J. Nucl. Phys. 32 (1980) 635; 33 (1981) 847.

[19] C. Timmermans for the L3 + C Collaboration, in: Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, Utah, USA, 17–25 August, 1999, Vol. 2, p. 9.

[20] C. Grupen et al., in: Proc. 25th Intern. Cosmic Ray Conf., Durban, South Africa, 28 July – 8 August, 1997, Vol. 7, p. 249.