PHYS6011 Experimental Problem Set SOLUTIONS

Part I

1. The Lorentz Force on a charged particle is

$$\mathbf{F} = qe(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The magnetic part of this provides the centripetal force causing a particle to move in a circle, so in the non-relativistic case

$$\begin{aligned} \frac{mv^2}{R} &= eqvB \\ \Rightarrow p &= eqBR \end{aligned}$$

(for relativistic particles $m \to \gamma m$ and this still holds). This is still in SI (or any other consistent) units so the momentum is in kg m s^{-1}, not in GeV/c.

$$\begin{aligned} 1 \text{ GeV/c} &= \frac{10^9 \times |e| \text{ J}}{c} \\ &= \frac{10^9 \times |e| \text{ kg m}^2 \text{ s}^{-2}}{3.0 \times 10^8 \text{ m s}^{-1}} \\ &= \frac{|e| \text{ kg m s}^{-1}}{0.3} \\ \Leftrightarrow 1 \text{ kg m s}^{-1} &= 0.3/|e| \text{ GeV/c} \\ \Rightarrow eqBR \text{ kg m s}^{-1} &= \frac{0.3}{|e|} eqBR \text{ GeV/c} \\ \Rightarrow p &= 0.3qBR \text{ GeV/c} \end{aligned}$$

2. (a) In the correct units to use the equation $p = 0.3qBR$: $p = 7000$ GeV, $q = +1$ and $R = 27000/2\pi = 4,300$. Thus

$$B = \frac{p}{0.3R} = 5.4 \text{ T}$$

(NB In fact there are straight sections included in the 27 km total, so the effective bending radius is actually smaller and the field required over 8 T).

(b) For LEP, the only difference is $p = 100$ GeV/c $\Rightarrow B = 0.078$ T.

(c) Power loss is given by

$$P = \frac{1}{6\pi\epsilon_0}\frac{e^2 v^4}{c^3 R^2}\gamma^4$$

but this is energy lost per unit time, so it must be multiplied by the time per revolution $= 2\pi R/v$. Working in SI units, and plugging in $v \approx 1$ and $\gamma = E/mc^2 = 2.0 \times 10^5$

$$\begin{aligned} \Delta E &= \frac{e^2\gamma^4}{3\epsilon_0 R} \\ &= \frac{(1.6 \times 10^{-19})^2(2.0 \times 10^5)^4}{3 \times 8.9 \times 10^{-12} \times 4,300} \\ &= 3.6 \times 10^{-10} \text{ J} \\ &= 2,200 \text{ MeV} \end{aligned}$$

(d) Peak accelerating fields are 10s of MV/m, so assuming a rough average figure of 10 MV/m, LEP would have required about 200 m of cavities.

3. (a) Protons in the beam have a four-momentum

$$p_1 = (E_{beam}, p, 0, 0) = (\sqrt{p^2 + m_p^2}, p, 0, 0)$$

Stationary hydrogen nuclei have four-momentum

$$p_2 = (m_p, 0, 0, 0)$$

Then

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ &= (E_{beam} + m_p)^2 - p^2 \\ &= p^2 + m_p^2 + 2E_{beam} m_p + m_p^2 - p^2 \\ &\approx 2pm_p \\ \Rightarrow \sqrt{s} &= \sqrt{2 \times 500 \times 1} \\ &= 30 \text{ GeV} \end{aligned}$$

(b) The four-momenta in a collider would be

$$p_1 = (E_{beam}, p, 0, 0)$$

and

$$p_2 = (E_{beam}, -p, 0, 0)$$

so

$$s = (p_1 + p_2)^2 = 4E_{beam}^2$$

or $\sqrt{s} = 2E_{beam}$ as expected, so for $\sqrt{s} = 30$ GeV, the required beam energy is only 15 GeV.

4. (a) Current is total charge per unit time. Since each proton or antiproton goes round the ring in $\Delta t = 2\pi r/c \approx 21\ \mu$s, the current is

$$I = \frac{(N_p + N_{\overline{p}}) \times |e|}{\Delta t} = \frac{9.0 \times 10^{12} \times 1.6 \times 10^{-19}}{21 \times 10^{-6}} = 70 \text{ mA}$$

(b) Total energy is 0.98 TeV$\times(7.9+1.1)\times10^{12} = 9\times10^{24}$ eV$= 1.4$ MJ.

(c) $\mathcal{L} = f n_1 n_2/A$, with $f = 36/\Delta t$ since there are 36 bunches. So

$$A = \frac{36 \times 7.9 \times 10^{12} \times 1.1 \times 10^{12}}{22 \times 10^{-6} \times 54 \times 10^{30}} = 0.3 \text{ cm}^2$$

(Note: since the density of particles in the beam is not uniform, the "luminous" region where most collisions occur is much smaller than this).

(d) One week of running is $7\times22 = 154$ hours$= 5.5\times10^5$ s. Integrated luminosity is therefore given by

$$\begin{aligned}\int \mathcal{L}dt &= 5.5 \times 10^5 \times 54 \times 10^{30} \\ &= 3.0 \times 10^{37} \text{ cm}^{-2} \\ &= 3.0 \times 10^{13} \text{ barn}^{-1} \\ &= 30 \text{ pb}^{-1}\end{aligned}$$

5. (a) Nitrogen has a density of about 1 g/litre so

$$\begin{aligned}dE/dx \approx 1.5 \times 0.001 &= 0.0015 \text{ MeV/cm} \\ &= 0.15 \text{ MeV/m}\end{aligned}$$

(b) The density of steel is 7.9 $\mathrm{g/cm^3}$ so

$$dE/dx \approx 1.5 \times 7.9 = 12 \text{ MeV/cm}$$

or 1.2 GeV/m. If the muon stayed MIP-like until it stopped it would travel $10/1.2 \approx 8$ m.

(The Bethe-Block curve starts rising as $1/v^2$ below about 1 GeV so this is an overestimate. However a 10 GeV muon will reach this point and stop being a MIP after losing 9 GeV in $9/1.2 = 7.5$ m, so the correction is small).

6. The sensitive region of a detector element is Δy wide, so an element centred at $y = 0$ will register hits from the region $-\Delta y/2 < y < \Delta y/2$. There is an equal probability of a hit occuring anywhere in this region, so

$$\begin{aligned}
\text{resolution} &= \sqrt{\langle y^2\rangle - \langle y\rangle^2} \\
&= \left(\frac{1}{\Delta y}\int_{-\Delta y/2}^{\Delta y/2} y^2 dy - 0\right)^{1/2} \\
&= \left(\frac{1}{\Delta y}\left[\frac{y^3}{3}\right]_{-\Delta y/2}^{\Delta y/2}\right)^{1/2} \\
&= \Delta y/\sqrt{12}
\end{aligned}$$

7. LEP was an electron-positron collider with bunch crossings every 22 μs, and since electron-positron collisions are relatively "clean" the number of particles per event traversing the chamber and leaving tracks was relatively low. With a typical drift speed of 5 cm/μs, the maximum potential overlap was only a few events (in the time it takes the charge to drift the maximum of 4 m, only 7 more collisions could have taken place). In fact, the cross section at LEP was low enough that the probability of overlapping events was negligible.

   The LHC by contrast will be a proton-proton collider with a 25 ns spacing between bunch crossings. This means a TPC would have hundreds of overlapping "messy" proton-proton collisions with many tracks (the large proton-proton cross section will actually produce multiple collisions per bunch crossing and make things far worse).

   The weaknesses of a TPC are the long drift time and the possibility of breakdown with large numbers of particles. Both of these are many orders of magnitude worse at the LHC than at LEP, so it would not be a suitable technology.

8. (a)

$$\begin{aligned} d &\approx 0.5\sqrt{\rho(V+0.5)} \\ \Rightarrow V &\approx 4d^2/\rho - 0.5 \\ &= 4 * 300^2/5000 - 0.5 \\ &\approx 70 \text{ V} \end{aligned}$$

(b) Assume $dE/dx = 1.5$ MeVg^{-1}cm^2 and the density of silicon is 2.3 g cm^{-3}, so in 0.03 cm $\Delta E = 1.5 * 2.3 * 0.03 = 0.1$ MeV.

(c) Most likely number of electron-hole pairs in 300 μm silicon is 22,000. Most likely energy deposit is less than the mean calculated in 8b, but for a quick estimate 100 keV is OK, so energy per pair $\approx 100,000/22,000 \approx 4.5$ eV (true value is 3.6 eV).

1.1 eV goes into the potential energy of each "freed" electron, promoting it into the conduction band. These electrons remain bound in the silicon crystal however, so momentum has to be conserved by the production of lattice vibrations (phonons) which account for the extra energy required.

9. (a)

$$\begin{aligned} X_0 &= \frac{716.4 \text{ g cm}^{-2}\ A}{Z(Z+1)\ln(287/\sqrt{Z})} \\ &= \frac{716.4 \times 184}{74 \times 75 \times \ln(287/\sqrt{74})} \\ &= 6.8 \text{ g cm}^{-2} \\ &= 3.5 \text{ mm} \end{aligned}$$

(b) After traversing a depth of nX_0, the average energy of a particle in this toy model is $E_0/2^n$. The shower stops when this average energy is less than 8.3 MeV

$$\Rightarrow 2^n < 100,000/8.3 = 12,000$$
$$\Rightarrow n \approx 14$$

This corresponds to $14 \times 3.5 = 50$ mm of tungsten or about 10 mm total calorimeter thickness, since the scintillator contributes negligibly to the showering.

(c) Again using the toy model, in the nth generation there are 2^n particles, so the total particle count summed over all generations is

$$\sum_{n=0}^{13} 2^n = \frac{2^{14}-1}{2-1} = 16000$$

where the normal geometric progression sum formula has been used.

Since each generation travels through one radiation length which contains on average 3.5 mm of scintillator, and for a MIP

$$dE/dx = 1.5 \times 1.1 = 1.7 \text{ MeV/cm}$$

the total ionisation energy released will be

$$\approx 16000 \times 0.35 \times 1.7 \approx 9000 \text{ MeV}$$

about one tenth of the incident energy.

(Since about 1/3 of the shower particles are photons, a correction factor could be applied and 6000 MeV is probably a more reasonable figure).

(d) A typical scintillator generates about 1 photon/100 eV, so

$$9 \times 10^9/100 = 9 \times 10^7$$

photons will be produced, of which 5% are detected, i.e. about 4.5×10^6

(e) Once again the scintillator can be ignored. The nuclear interaction length of tungsten is $\lambda \approx 35A^{1/3} \approx 200$ g cm^{-2} ≈ 10 cm, so a calorimeter with 5 cm of tungsten is only about 0.5λ thick.

10. (a) The radiation length of aluminium is 8.9 cm, so

$$\theta_{rms} \approx \frac{13.6\text{MeV}}{\beta cp} q\sqrt{x/X_0} = \frac{1.4\text{MeV}q}{\beta cp}$$

(b) For a 500 MeV pion, this becomes $\theta_{rms} \approx 1.4/500 \approx 3\times10^{-3}$ radians. Extrapolating over 6 cm gives an RMS error of $6\times3\times10^{-3}$ cm ≈ 180 μm.