PHYS6011 Experimental Problem Set SOLUTIONS

Part II

1. The frequency of an event occurring is given by

$$f = \mathcal{L}\sigma$$

so for $b\bar{b}$ production with $\sigma \sim 10\ \mu$b

$$f = 50 \times 10^{30} \times 10 \times 10^{-6} \times 10^{-24} = 500\ \text{s}^{-1}$$

Similarly a W is produced with a frequency of 0.5 s^{-1} or one every 2 s, and a top quark pair with a frequency of $5 \times 10^{-4}\ \text{s}^{-1}$ which is about once every half hour.

2. In SI units: $\tau = \hbar/\Gamma \approx 4 \times 10^{-25}$ s. Hadronisation takes place at a scale of $\Lambda_{QCD} \approx 200$ MeV corresponding to a length scale of 1 fm (remembering that $\hbar c \approx 200$ MeV fm) and a time scale of 1 fm/$c \approx 3 \times 10^{-24}$ s. This means that top quarks will decay approximately ten times faster than hadrons will form.

   Notice that in natural units time and distance are both measured in GeV^{-1}, so the top lifetime is just $1/1.5 \approx 0.7\ \text{GeV}^{-1}$.

3. Since $Br(W \to l\nu) = 10\%$ per lepton, 20% of top decays will be to an electron or muon. The probability of an event with both tops decaying this way is then $0.2 \times 0.2 = 4\%$.

4. (a) The four momentum of a particle with energy E traveling in a direction in spherical coordinates (θ, ϕ) is

$$(E, p_x, p_y, p_z) = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta)$$

where the magnitude of the 3-momentum for a particle with mass m is $p = \sqrt{E^2 - m^2}$. This gives the four momenta (in GeV):

$$\begin{aligned} b_1 &: (145, -1, -43, 139) \\ b_2 &: (90, 85, -17, 24) \\ j_1 &: (125, -49, 85, 78) \\ j_2 &: (15, -2, -15, -2) \end{aligned}$$

(b) By four momentum conservation, the sum of the momenta of j_1 and j_2 gives the momentum of the parent X in the decay $X \to j_1 j_2$, so

$$p_X = (140, -51, 70, 76) \text{ GeV}$$

and

$$\begin{aligned} m_X^2 = p_X^2 &= 140^2 - 51^2 - 70^2 - 76^2 \\ \Rightarrow m_X &= 80 \text{ GeV} \end{aligned}$$

This is very close to the W mass, so it is likely that the two jets were produced by the quarks in a $W \to q\bar{q}$ decay.

(c) We can calculate the invariant mass of the top in a similar fashion for the possible $t \to Wb$ decays. Adding the b_1 four momentum to that of the W from the previous section gives

$$p_1 = (285, -52, 27, 216) \Rightarrow m_1 = 176 \text{ GeV}$$

Combining the W and b_2 gives

$$p_2 = (230, 34, 53, 100) \Rightarrow m_2 = 197 \text{ GeV}$$

The first mass is consistent with the top mass, so it is probable that b_1 and the two light quark jets come from the same top decay.

5. (a) The total error, σ_{tot} is obtained by adding the systematic and background statistical error ($= \sqrt{N_B}$) in quadrature, so

$$S = \frac{N - N_B}{\sigma_{tot}} = \frac{30 - 17}{\sqrt{5^2 + 17}} = 2.0$$

(b) In a total data sample of $n \times 200$ pb^{-1} we can predict $n \times 30$ total events and $n \times 17$ background events, so for a discovery

$$S = 5 = \frac{n \times (30 - 17)}{\sqrt{5^2 + n \times 17}}$$

$$\begin{aligned} \Leftrightarrow 5\sqrt{25 + 17n} &= 13n \\ \Rightarrow 169n^2 - 425n - 625 &= 0 \end{aligned}$$

Solving the quadratic gives a requirement of $n = 3.6$, or an extra $2.6 \times 200 = 520$ pb^{-1}.