PHYS6011 Experimental Problem Set 2008
Lecture 3 - Answers

1. The frequency of an event occurring is given by the product of the luminosity and the cross-section:
$$f = \mathcal{L}\sigma$$
so for $b\bar{b}$ production with $\sigma = 10\,\mu$b:
$$f = 1.1 \times 10^{34}\,\mathrm{cm}^{-2} s^{-1} \times 10 \times 10^{-6} \times 10^{-24}\,\mathrm{cm}^2 = 1.1 \times 10^4\,\mathrm{s}^{-1}$$
Repeating for the others the rate is b) $11\,\mathrm{s}^{-1}$; and c) $0.011\,\mathrm{s}^{-1}$

2. In SI units, the lifetime $\tau = \hbar/\Gamma = 4 \times 10^{-25}\,$s. Hadronisation takes place at a scale of $\Lambda_{QCD} \approx 200\,$MeV corresponding to a length scale of 1 fm (remembering that $\hbar c \approx 200\,$MeV fm) and a time scale of $1\,fm/c \approx 3 \times 10^{-24}\,$s. This means that top quarks will decay approximately 10 times faster than the hadrons will form.

   Notice that in natural units, time and distance are both measured in GeV^{-1}, so the top lifetime is just $1/1.5 \approx 0.7\,\mathrm{GeV}^{-1}$.

3. Since the branching fraction BF($W \to l\nu$) = 10% per lepton, 20% of top decays will be to an electron or muon. The probability of an event with both tops decaying to an electron or muon is then $0.2 \times 0.2 = 4\%$.

4. The four momentum of a particle with mass m, energy E and momentum p ($E^2 = p^2 + m^2$) travelling in a directions in spherical coordinates (θ, ϕ) is:
$$(E, p_x, p_y, p_z) = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta)$$
   (a) This gives the four-momenta of the 4 particles as:
   - $b_1 : (E, \bar{p}) = (145, -1, -43, 139)$
   - $b_2 : (E, \bar{p}) = (90, 85, -17, 24)$
   - $j_1 : (E, \bar{p}) = (125, -49, 85, 78)$
   - $j_2 : (E, \bar{p}) = (15, -2, -15, -2)$

   (b) Four-momentum conservation requires the sum of the four-momenta of j_1 and j_2 gives the four-momentum of the parent in the decay $X \to j_1 j_2$ so:
$$p_X = (140, -51, 70, 76)$$
   and, using $E_X^2 = p_X^2 + m_X^2$, gives
$$\begin{aligned} m_X^2 = p_X^2 &= 140^2 - (-51)^2 - 70^2 - 76^2 \\ \Rightarrow m_X &= 80\,\mathrm{GeV} \end{aligned}$$
   This is very close to the W mass, so it is likely that the two jets were produced by quarks in a $W \to q\bar{q}$ decay.

   (c) We can calculate the invariant mass of the top in a similar fashion for the possible $t \to W^+ b$ decays. Adding the b_1 four-momentum to that of the W from the previous section gives:
$$p_1 = (285, -52, 27, 216) \Rightarrow m_1 = 176\,\mathrm{GeV}/c^2$$

Combining the W and the other b_2 gives

$$p_2 = (230, 34, 53, 100) \Rightarrow m_2 = 197\,\mathrm{GeV}/c^2$$

The first mass is consistent with the top mass ($\sim 175\,\mathrm{GeV}/c^2$), so it is probable that the b_1 and the two light quark jets come from the same top decay.

5. (a) The total error, σ_{tot}, is obtained by adding the systematic and the background error ($= \sqrt{N_B}$) in quadrature:

$$S = \frac{N_S - N_B}{\sigma_{tot}} = \frac{30 - 17}{\sqrt{5^2 + 17}} = 2.0$$

(b) In a total data sample of $n \times 200\,\mathrm{pb}^{-1}$ we can predict that there will be $30 \times n$ events and $17 \times n$ background events, so for a discovery:

$$\begin{aligned} S = 5 &= \frac{n \times (30 - 17)}{\sqrt{5^2 + 17 \times n}} \\ \Rightarrow 5\sqrt{25 + 17n} &= 13n \\ \Rightarrow 169n^2 - 425n - 625 &= 0 \end{aligned}$$

solving the quadratic equation gives the requirement that $n = 3.6$ or an extra $2.6 \times 200 = 520\,\mathrm{pb}^{-1}$.

6. (a) The Gaussian distribution has a width $\sigma = 0.4$ and a mean $\mu = 0.6$ (in units of 10^{-6}). So the probability that the result is less than zero is:

$$\int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.0668 \qquad \text{(from table of normal distributions)}$$

So there is a 6.7% probability that another measurement would produce a negative answer.

An alternative way to look at this is that the mean is 1.5σ away from zero so the probability being less than 0 is 1 less the integral of the Normal distribution from $-\infty$ to 1.64 (because the Normal distribution is symmetric about 0) so probability = (1-0.9332) = 0.0668.

(b) We want a number that ensures that the integral of Gaussian distribution is equal 0.95:

$$\int_{-\infty}^{X} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.95$$

The integral equals 0.95 when X equals $\mu + 1.64\sigma$. So X must equal $(0.6 + 0.4 \times 1.64) \times 10^{-6} = 1.26 \times 10^{-6}$.