

1. The Lorentz Force on a charged particle is

$$\mathbf{F} = qe(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The magnetic part of this provides the centripetal force causing a particle to move in a circle so in the non-relativistic case:

$$\begin{aligned} \frac{mv^2}{R} &= eqvB \\ \Rightarrow p &= eqBR \end{aligned}$$

For relativistic particles $m \rightarrow \gamma m$ and the equation still holds. This is still in SI units so the momentum is in $kg m s^{-1}$. We want it in GeV/c :

$$\begin{aligned} 1 \text{ GeV}/c &= \frac{10^9 \times |e|J}{c} \\ &= \frac{10^9 \times |e| kg m^2 s^{-2}}{3 \times 10^8 m s^{-1}} \\ &= \frac{|e| kg m s^{-1}}{0.3} \\ \Rightarrow 1 kg m s^{-1} &= \frac{0.3}{|e|} \text{ GeV}/c \\ \Rightarrow eqBR kg m s^{-1} &= \frac{0.3eqBR}{|e|} \text{ GeV}/c \\ \Rightarrow p &= 0.3qBR \text{ GeV}/c \end{aligned}$$

2. (a) In a cyclotron $p \text{ (GeV}/c) = 0.3qBR$. Setting $p = 104 \text{ GeV}/c, q = 1, R = 27000/2\pi = 4.3 \times 10^3 m$ gives:

$$B = \frac{p}{0.3qR} = \frac{100}{0.3 \times 4300} = 0.081 T$$

(b) For LHC, $p = 7000 \text{ GeV}/c$ and so $B = 5.4 T$.

(c) The energy is related to the speed through the relativistic equation $E = \gamma mc^2$. Using reduced units by dropping the c^2 , this is $E = \gamma m$. Setting the proton mass $m_p \approx 1 \text{ GeV}$ gives $\gamma = 7 \text{ TeV}/1 \text{ GeV} = 7000$. Therefore:

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \beta^2}} \\ \Rightarrow \beta &= \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \\ \Rightarrow \beta &= 0.999999989c \end{aligned}$$

(d) The power loss in Watts (energy lost per second) is given by:

$$P = \frac{1}{6\pi\epsilon_0} \frac{e^2 v^4 \gamma^4}{c^3 R^2}$$

To turn this into energy lost per turn we must multiply by the time for one revolution $= 2\pi R/v$. Working in SI units, and setting $v \approx c$ and $\gamma = E/m = 7000$:

$$\begin{aligned} \Delta E \text{ (Energy lost per turn)} &= \frac{e^2 \gamma^4}{3\epsilon_0 R} \\ &= \frac{(1.6 \times 10^{-19})^2 (7000)^4}{3 \times 8.9 \times 10^{-12} \times 4.3 \times 10^3} \\ &= 5.4 \times 10^{-16} \text{ J/turn} \\ &= 3.3 \text{ keV/turn} \end{aligned}$$

(e) The number of revolutions per second is $= v/2\pi R = 1.1 \times 10^4$. Therefore the energy lost per proton per second $= 3.3 \text{ keV/turn} \times 1.1 \times 10^4 \text{ turns} = 37 \text{ MeV/s}$. Therefore energy lost for all protons in all bunches is $= 37 \times 10^6 \text{ eV} \times 2802 \times 10^{11} \times 1.6 \times 10^{-19} \text{ J/eV} = 1.6 \text{ MWatts}$. This is the energy that has to be replaced each second.

(f) Annual electricity bill is $1.6 \text{ MW} \times 2$ (two beams) $= 3.2 \text{ M€}$. In fact the electricity bill is much higher.

3. The length of the accelerator is given by:

$$\begin{aligned} l &= \frac{\text{(Final Energy)}}{\text{(Accelerating Field per metre)}} \\ &= \frac{500 \text{ GeV}}{10 \text{ MV/m}} = 50 \text{ km} \end{aligned}$$

4. (a) Using 4-momenta notation, $p = (E, \vec{p}) = (E, p_x, p_y, p_z)$, and Einstein's equation $E^2 = m^2 + p^2$ in natural units, the initial 4-momenta are (we could equally have had the particles travelling along the x or y direction):

$$\begin{aligned} p_1 &= (E_1, 0, 0, p) = (\sqrt{p^2 + m_p^2}, 0, 0, p) && \text{Beam} \\ p_2 &= (E_2, 0, 0, 0) = (m_p, 0, 0, 0) && \text{Target} \\ \Rightarrow s &= (p_1 + p_2)^2 \\ &= (E_1 + E_2)^2 - p^2 \\ &= (E_1 + m_p)^2 - p^2 \\ &= (500.001 + 1)^2 - 500^2 \\ \Rightarrow \sqrt{s} &\approx 31 \text{ GeV} \end{aligned}$$

(b) The 4-momenta in a collider would be:

$$\begin{aligned} p_1 &= (E_{beam}, 0, 0, p) \\ p_2 &= (E_{beam}, 0, 0, -p) \end{aligned}$$

$$\text{Therefore } s = (p_1 + p_2)^2 = 4E_{beam}^2$$

So to achieve $\sqrt{s} = 31 \text{ GeV}$, the beam energy only needs to be 15.5 GeV.

5. (a) Current is total charge per unit time. Each proton and anti-proton goes round the ring in time $\Delta t = 2\pi R/c \approx 90 \mu\text{s}$. Therefore the current is:

$$I = \frac{(N_p + N_{\bar{p}}) \times |e|}{\Delta t} = \frac{2 \times 2808 \times 10^{11} \times 1.6 \times 10^{-19}}{90 \times 10^{-6}} = 1.0 \text{ Amps}$$

The actual average current is ~ 0.6 Amps as the number of particles in the bunches decreases with time (as does the luminosity).

- (b) The total energy is $7 \text{ TeV} \times 2808 \times 2 \times 10^{11} = 7.85 \times 10^{27} \text{ eV} = 600 \text{ MJ}$.
(c) Luminosity $\mathcal{L} = fn_1n_2/A$ with $f = 2808/\Delta t$. So:

$$\text{Area} = \frac{fn_1n_2}{\mathcal{L}} = \frac{2808 \times 10^{11} \times 10^{11}}{90 \times 10^{-6} \text{ s} \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}} = 3 \times 10^{-5} \text{ cm}^2$$

Since the density of particles in the beam is not uniform the “luminous” region where most of the collisions is much smaller than this.

- (d) One month of running is $30 \text{ days} \times 23 \text{ hrs/day} = 2.5 \times 10^6 \text{ s}$. Therefore, the integrated luminosity:

$$\begin{aligned} \int_0^{2.5 \times 10^6} \mathcal{L} dt &= 2.5 \times 10^6 \times 10^{34} \text{ cm}^{-2} \\ &= 2.5 \times 10^{40} \text{ cm}^{-2} \\ &= 2.5 \times 10^{16} \text{ barn}^{-1} \\ &= 25 \text{ fb}^{-1} \end{aligned}$$

- (e) Number of Higgs seen is the integrated luminosity times the cross-section times the efficiency:

$$N = 25 \text{ fb}^{-1} \times 2 \text{ pb} \times 0.02 = 500$$