1. (a) The mean energy loss is:

$$\mathrm{d}E/\mathrm{d}x = 1.5\,\mathrm{MeV\,g^{-1}\,cm^2} \times 10^{-3}\,\mathrm{g\,cm^{-3}} = 0.0015\,\mathrm{MeV/\,cm} = 0.15\,\mathrm{MeV/\,m}$$

   (b) For steel, $\mathrm{d}E/\mathrm{d}x = 1.5\,\mathrm{MeV\,g^{-1}\,cm^2} \times 18.95\,\mathrm{g\,cm^{-3}} = 2.8\,\mathrm{GeV/m}$. So if the muon stayed MIP-like, it would travel a distance $10\,\mathrm{GeV}/2.8\,\mathrm{GeV}/m = 3.6\,\mathrm{m}$ before stopping. In reality, the energy loss of the Bethe-Block curve starts to rise as v^{-2} below about 1 GeV. However a 10 GeV muon will continue to act like a MIP until this point and will have travelled $9\,\mathrm{GeV}/2.8\,\mathrm{GeV/\,m} = 3.2\,\mathrm{m}$ so the correction is small (at most $0.3\,\mathrm{m}$).

2. The sensitive region of a detector element is Δy wide so an element centred at $y = 0$ will register hits from the region $-\Delta y/2 < y < \Delta y/2$. There is an equal probability of a hit occurring anywhere in this region, so from a standard statistics book:

$$\begin{aligned}
\text{resolution} &= \sqrt{\langle y^2\rangle - \langle y\rangle^2} \\
&= \left(\frac{1}{\Delta y}\int_{-\Delta y/2}^{\Delta y/2} y^2 dy - 0\right)^{1/2} \\
&= \left(\frac{1}{\Delta y}\left[\frac{y^3}{3}\right]_{-\Delta y/2}^{\Delta y/2}\right)^{1/2} \\
&= \Delta y/\sqrt{12}
\end{aligned}$$

3. LEP was an electron-positron collider with bunch crossings every 22 μs and since electron-positron collisions are relatively "clean" the number of particles per event traversing the chamber and leaving tracks was relatively low. With a typical drift speed of $5\,\mathrm{cm}/\,\mu\mathrm{s}$, the maximum potential overlap was only a few events (in the time to drift 4 m only 7 more collisions could have taken place). In fact, the cross-section at LEP was low enough that the probability of overlapping events was negligible.

   The LHC by contrast will be a proton-proton collider with a 25 ns spacing between bunch crossings. This means a TPC would have hundreds of overlapping "messy" proton-proton collisions with many tracks. The cross-section for proton-proton collisions is also larger so there is a higher probability of each bunch crossing having a collision (as high as 20 collisions per bunch crossing).

   The weaknesses of a TPC are the long drift time and the possibility of breakdown with large number of particles. Both of these are many magnitudes worse at the LHC than at LEP so it would not be a suitable technology.

4. (a) The size of the depletion layer under a reverse bias is:

$$\begin{aligned}
d\,\mathrm{cm} &\approx 0.5\sqrt{\rho(V+0.5)} \\
\Rightarrow V &\approx 4d^2/\rho - 0.5 \\
&= 4 \times 200^2/5000 - 0.5 \\
&\approx 31.5V
\end{aligned}$$

   (b) If $\mathrm{d}E/\mathrm{d}x = 1.5\,\mathrm{MeV\,g^{-1}\,cm^2}$, so in $0.02\,\mathrm{cm}$ the energy loss is $1.5\,\mathrm{MeV\,g^{-1}\,cm^2} \times 2.3\,\mathrm{g\,cm^{-3}} \times 0.02\,\mathrm{cm} = 0.69\,\mathrm{MeV}$.

(c) The number of electron-hole pairs in 200 μm thick silicon is $= 7.5\times10^7\times200\times10^{-6} = 15,000$. The total energy deposited (on average) is $\Delta E = 0.69\,\mathrm{MeV}$, so the energy needed to liberate each electron-hole pair is $= 0.69\,\mathrm{MeV}/15000 \approx 4.6\,\mathrm{eV}$. The true value is $3.6\,\mathrm{eV}$.

1.1 eV goes into the potential energy of each "freed" electron, promoting it to the conduction band. These electrons remain bound in the silicon crystal however so momentum has to be conserved by the production of lattice vibrations (phonons) which account for the extra energy needed.

5. (a) The radiation length X_0 ($\mathrm{g\,cm^{-2}}$) is given by:

$$
\begin{aligned}
X_0\ (\mathrm{g\,cm^{-2}}) &= \frac{716.4\,\mathrm{g\,cm^{-2}A}}{Z(Z+1)\ln(287/\sqrt{Z})} \\
&= \frac{716.4\times184}{74\times75\times\ln(287/\sqrt{74})} \\
&= 6.8\,\mathrm{g\,cm^{-2}} \\
\Rightarrow X_0(\mathrm{cm}) &= \frac{6.8\,\mathrm{g\,cm^{-2}}}{19.5\,\mathrm{g\,cm^{-3}}} = 3.5\,\mathrm{mm}
\end{aligned}
$$

(b) On average a particle will lose half its energy every X_0 (because it creates 2 new particles). After traversing n radiation lengths, the number of particles will be 2^n in the nth generation, so the average energy per particle will be $E_0/2^n$. So the shower will stop when this average energy is less than the critical energy 8.3 MeV.

$$
\begin{aligned}
\Rightarrow &\quad \frac{100\,\mathrm{GeV}}{2^n} < 8.3\,\mathrm{MeV} \\
\Rightarrow &\quad n \approx 14
\end{aligned}
$$

Therefore the depth is $14\times3.5\,\mathrm{mm} = 50\,\mathrm{mm}$ of Tungsten or about 10 cm of total calorimeter thickness, since the scintillator contributes negligibly to the showering.

(c) In the nth generation, there are 2^n particles so the total number of particles created is:

$$\sum_{n=0}^{13} 2^n = \frac{2^{14}-1}{2-1} = 16000$$

where the normal geometric progression sum formula has been used. Since each generation travels through one radiation length which contains on average 3.5 mm of scintillator and $\mathrm{d}E/\mathrm{d}x = 1.5\,\mathrm{MeV\,g^{-1}\,cm^2}\times1.1\mathrm{g\,cm^{-3}} = 1.7\,\mathrm{MeV/\,cm}$, the total ionisation energy released will be:

$$16000\times0.35\,\mathrm{cm}\times1.7\,\mathrm{MeV/\,cm} = 9\,\mathrm{GeV}$$

This is about one-tenth the incident energy so we only see a fraction of the energy deposited in the Tungsten. Also, about 1/3rd of the shower particles are photons so 6 GeV is probably a more reasonable figure.

(d) A typical scintillator generates about 1 photon per 100 eV so:

$$
\begin{aligned}
\text{Number of photons produced} &= \frac{9\,\mathrm{GeV}}{100\,\mathrm{eV}} = 9\times10^7 \\
\text{Number of photons detected} &= 9\times10^7\times0.05 = 4.5\times10^6
\end{aligned}
$$

(e) Ignoring the scintillator, the nuclear interaction length (λ_{int}) of Tungsten is:

$$\lambda \approx 35A^{1/3} = 200\,g\,\mathrm{cm}^{-2} = 10\,\mathrm{cm}$$

So a calorimeter with 5 cm of Tungsten is only 0.5λ thick. This is why hadron calorimeters have to be larger (deeper) than electro-magnetic calorimeters.

6. (a) The radiation length of aluminium is 8.9 cm, so

$$\theta_{rms} \approx \frac{13.6\,\mathrm{MeV}}{\beta cp} q \sqrt{\frac{x}{X_0}} = \frac{1.4\,\mathrm{MeV}q}{\beta cp}$$

(b) For a 500 MeV pion, this becomes $\theta_{rms} = 1.4/500 \approx 3 \times 10^{-3}$ rads. Extrapolating back over 6 cm gives an RMS error of $6 \times 3 \times 10^{-3}\,\mathrm{cm} \approx 180\,\mu\mathrm{m}$. This is why a lot of effort is made to keep detectors as thin as possible.