PHYS6011 Experimental Particle Physics - Problem Set 3 Answers - 2014

1. The frequency of an event occurring is given by the product of the luminosity and the cross-section:

$$f = \mathcal{L}\sigma$$

so for $b\bar{b}$ production with $\sigma = 10\,\mu$b:

$$f = 1.1 \times 10^{34}\,\mathrm{cm}^{-2}s^{-1} \times 10 \times 10^{-6} \times 10^{-24}\,\mathrm{cm}^2 = 1.1 \times 10^4\,\mathrm{s}^{-1}$$

Repeating for the others the rate is b) $11\,\mathrm{s}^{-1}$; and c) $0.011\,\mathrm{s}^{-1}$

2. In SI units, the lifetime $\tau = \hbar/\Gamma = 4 \times 10^{-25}\,$s. Hadronisation takes place at a scale of $\Lambda_{QCD} \approx 200\,$MeV corresponding to a length scale of 1 fm (remembering that $\hbar c \approx 200\,$MeV fm) and a time scale of $1\,fm/c \approx 3 \times 10^{-24}\,$s. This means that top quarks will decay approximately 10 times faster than the hadrons will form.

   Notice that in natural units, time and distance are both measured in GeV^{-1}, so the top lifetime is just $1/1.5 \approx 0.7\,\mathrm{GeV}^{-1}$.

3. Since the branching fraction BF$(W \to l\nu) = 10\%$ per lepton, 20% of top decays will be to an electron or muon. The probability of an event with both tops decaying to an electron or muon is then $0.2 \times 0.2 = 4\%$.

4. The four momentum of a particle with mass m, energy E and momentum p ($E^2 = p^2 + m^2$) in spherical coordinates (θ, ϕ) is:

$$(E, p_x, p_y, p_z) = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta)$$

   (a) This gives the four-momenta of the 4 particles as:

   - $b_1 : (E, \bar{p}) = (145, -1, -43, 139)$
   - $b_2 : (E, \bar{p}) = (90, 85, -17, 24)$
   - $j_1 : (E, \bar{p}) = (125, -49, 85, 78)$
   - $j_2 : (E, \bar{p}) = (15, -2, -15, -2)$

   (b) Four-momentum conservation requires the sum of the four-momenta of j_1 and j_2 gives the four-momentum of the parent in the decay $X \to j_1 j_2$ so:

$$p_X = (140, -51, 70, 76)$$

   and, using $E_X^2 = p_X^2 + m_X^2$, gives

$$\begin{aligned} m_X^2 = E_X^2 - p_X^2 &= 140^2 - ((-51)^2 + 70^2 + 76^2) \\ \Rightarrow m_X &= 83\,\mathrm{GeV} \end{aligned}$$

   This is very close to the W mass, so it is likely that the two jets were produced by quarks in a $W \to q\bar{q}$ decay.

   (c) We can calculate the invariant mass of the top in a similar fashion for the possible $t \to W^+ b$ decays. Adding the b_1 four-momentum to that of the W from the previous section gives:

$$p_1 = (285, -52, 27, 216) \Rightarrow m_1 = 176\,\mathrm{GeV}/c^2$$

Combining the W and the other b_2 gives

$$p_2 = (230, 34, 53, 100) \Rightarrow m_2 = 197\,\mathrm{GeV}/c^2$$

The first mass is consistent with the top mass ($\sim 175\,\mathrm{GeV}/c^2$), so it is probable that the b_1 and the two light quark jets come from the same top decay.

5. (a) The total error, σ_{tot}, is obtained by adding the systematic and the background error ($= \sqrt{N_B}$) in quadrature:

$$S = \frac{N_S - N_B}{\sigma_{tot}} = \frac{30 - 17}{\sqrt{5^2 + 17}} = 2.0$$

Therefore the result is about 2.0 standard deviations away from the null hypothesis (no signal events). This is too small to declare that this decay has been confidently measured.

(b) In a total data sample of $n \times 200\,\mathrm{pb}^{-1}$ we can predict that there will be $30 \times n$ events and $17 \times n$ background events, so for a discovery:

$$\begin{aligned} S = 5 &= \frac{n \times (30 - 17)}{\sqrt{5^2 + 17 \times n}} \\ \Rightarrow 5\sqrt{25 + 17n} &= 13n \\ \Rightarrow 169n^2 - 425n - 625 &= 0 \end{aligned}$$

solving the quadratic equation gives the requirement that $n = 3.6$ or an extra $2.6 \times 200 = 520\,\mathrm{pb}^{-1}$.

6. (a) The Gaussian distribution has a width $\sigma = 0.4$ and a mean $\mu = 0.6$ (in units of 10^{-6}). So the probability that the result is less than zero is:

$$\int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.0668 \qquad \text{(from a table of normal distributions)}$$

So there is a 6.7% probability that another measurement would produce a negative answer.

An alternative way to look at this is that the mean is 1.5σ away from zero so the probability of a measurement being less than 0 is 1 less the integral of the Normal distribution from $-\infty$ to 1.50 (because the Normal distribution is symmetric about 0) so probability = (1-0.9332) = 0.0668.

(b) We want a number that ensures that the integral of the Gaussian distribution is equal to 0.95:

$$\int_{-\infty}^{X} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.95$$

The integral equals 0.95 when X equals $\mu + 1.64\sigma$. So X must equal $(0.6 + 0.4 \times 1.64) \times 10^{-6} = 1.26 \times 10^{-6}$.